University of Bahrain

College of Information Technology
Department of Computer Science

ITCS253 Discrete Structures II

First Semester 2012/2013
Exam #2 — One Hour

STUDENT NAME	DRAGON
STUDENT#	
SECTION	

Answer All Questions.
Make sure you have 4 pages.

QUESTION#	MARKS		COMMENTS
1	3	3	
2	3	3	
3	8	8✓	
4	7	7✓	
5	7	7	
6	3	3✓	
7	6	6	
8	8	4✓	
TOTAL	45	~~37~~ 41	

Instructors: Dr. Ali Alsaffar.
Dr. Ali Khan (Coordinator).

Q1. [**3 marks**] Given the recurrence relation

$$2a_n - 5a_{n-1} + 6a_{n-2} = (3n+2)3^n$$

Write *Only* the characteristic equation.

$2a_n - 5a_{n-1} + 6a_{n-2} = 3n \cdot 3^n + 2 \cdot 3^n$

3 $(x^2 - 5x + 6)(x-3)^2(\cancel{x-3}) = 0$ ✓

Q2. [**3 marks**] For a linear homogeneous equation to solve $t(n)$, the roots of the characteristic equation are found as $r_1 = 2,\ r_2 = -1$, and $r_3 = 2$. Find expression of $t(n)$ in terms of some constants.

$t(n) = C_1 r_1^n + C_2 r_2^n + C_1 n r_3^n$

2 $= C_1 2^n + C_2(-1)^n + C_1 n (2)^n$

Q3. [**8 marks**] Use the method of substitution to solve the following recurrence relation.
$a_1 = 1,\ a_n = a_{n-1} + n,\ n \geq 2$

working relation:

$a_k = a_{k-1} + k$

For $k = n-1 \Rightarrow a_{n-1} = a_{n-2} + n - 1$

For $k = n-2 \Rightarrow a_{n-2} = a_{n-3} + n - 2$

For $k = n-3 \Rightarrow a_{n-3} = a_{n-4} + n - 3$

حل أسهل وأفضل

$a_n = a_{n-1} + n$
$= a_{n-2} + n - 1 + n$
$= a_{n-2} + 2n - 1$
$= a_{n-3} + n - 2 + 2n - 1$
$= a_{n-3} + 3n - 2 - 1$
$= a_{n-4} + n - 3 + 3n - 2 - 1$
$= a_{n-4} + 4n - 3 - 2 - 1$
$= a_{n-i} + in - (i-1) - (i-2) - \cdots - 3 - 2 - 1$

Let $n - i = 1 \Rightarrow n = 1 + i \Rightarrow i = n - 1$

$a_n = 1 + (\cancel{n-1})(n) - (i-1) - (i-2) - \cdots - 3 - 2 - 1$

multiplay both side by -1:

$-a_n = -1 - (n-1)(n) + (i-1) + (i-2) + \cdots + 3 + 2 + 1$

$-a_n = -1 - (n-1)(n) + \sum_{k=1}^{i-1} k$

8 $-a_n = -1 - (n-1)(n) + \frac{(i-1)(i)}{2}$

$-a_n = -1 - (n-1)(n) + \frac{(n-2)(n-1)}{2}$ $\quad \times -1$

$a_n = 1 + (n-1)(n) - \frac{(n-2)(n-1)}{2}$

Q4. [**7 marks**] Show that $C(n, r) = C(n-1, r) + C(n-1, r-1)$.

R.H.S:

$$= \frac{(n-1)!}{(n-1-r)!\,r!} + \frac{(n-1)!}{(n-r)!\,(r-1)!}$$

$$= \frac{(n-1)!}{(n-r-1)!\,r(r-1)!} + \frac{(n-1)!}{(n-r)(n-r-1)!\,(r-1)!}$$

$$= \frac{(n-1)!}{(n-r-1)!\,(r-1)!}\left[\frac{1}{r} + \frac{1}{n-r}\right]$$

$$= \frac{(n-1)!}{(n-r-1)!\,(r-1)!}\left[\frac{n-r+r}{r(n-r)}\right]$$

7

$$= \frac{(n-1)!}{(n-r-1)!\,(r-1)!} \cdot \frac{n}{r(n-r)}$$

$$= \frac{n!}{(n-r)!\,r!} = \text{L.H.S}$$

Q5. [**7 marks**] If $C(6, 2n-4) = C(6, 2n-6)$, find $C(n, 2)$.

$$C(6, 2n-4) = C(6, 6-2n+4)$$
$$= C(6, 10-2n)$$
$$\therefore C(6, 10-2n) = C(6, 2n-6)$$
$$\therefore 10-2n = 2n-6$$
$$4n = 16$$
$$n = 4$$

7

$$\therefore C(4,2) = \frac{4!}{2!\,2!} = \frac{4\times3\times2\times1}{4} = 3\times2 = 6$$

W M W M W M W M W
1 2 3 4 5 6 7 8 9

Q6. [**3 marks**] It is required to seat 4 men and 5 women in a row so that the men occupy even places. How many such arrangements are possible?

3

$P(4,4) \times P(5,5) = 4! \times 5!$ ✓

Q7. An instructor gives an exam with 14 questions. Students are allowed to choose any 10 to answer.

(a) [**3 marks**] How many different choices of ten questions are there?

3

$C(14,10) = \frac{14!}{4!\,10!}$ ✓

(b) [**3 marks**] Suppose 6 questions require proof and 8 do not. How many ten questions contain four that require proof and six that do not?

3

$C(6,4) \times C(8,6) = \frac{6!}{2!\,4!} \times \frac{8!}{2!\,6!}$ ✓

14
proof — not
6 — 8
4 — 10 — 6

Q8. [**8 marks**] Use the techniques for Inhomogeneous recurrence relations to solve the following.
$T(1) = 1,\ T(n) = 2 \cdot T(n/2) + n,\ n \geq 2$

Let $n = 2^i \Rightarrow i = \log_2 n$

$T(2^i) = 2.T(\frac{2^i}{2}) + 2^i$
$= 2.T(2^{i-1}) + 2^i$

Let $T_i = T(2^i) = T(n)$

$T_i = 2T_{i-1} + 2^i$
$T_i - 2T_{i-1} = 2^i$

Ch. eq:
$(x-2)(x-2)$
$r_1 = r_2 = 2$
$T_i = C_1 2^i + C_2 i 2^i$

$T(n) = C_1 n + C_2 . \log_2 n . n$

For $n=1 \Rightarrow T(1) = C_1(1) + C_2 \log_2 1 . n = 1$
$= \boxed{C_1 = 1}$ — (1)

from recursion:
For $n=2 \Rightarrow T(2) = 2.T(1) + 2$
$= 2(1) + 2 = 4$

$\therefore$ for $n=2 \Rightarrow T(2) = C_1(2) + C_2 . \log_2 2 . 2 = 4$
$= 2C_1 + 2C_2 = 4$ — (2)

subtituting (1) in (2):
$= 2 + 2C_2 = 4 \Rightarrow \boxed{C_2 = 1}$

$\therefore T(n) = n + n\log_2 n$

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